COLLEGE ALGEBRA - Samples

The following is a selection of samples drawn from the text, **COLLEGE ALGEBRA**.

A. From Chapter 2, Exponents and Radicals, pages 55-56.

📖 The ideas, concepts, principles, laws and applications of algebra are communicated through a kind of mathematical language. This written and spoken "language" of algebra involves, and is facilitated by the use of symbols. In order for a student to develop an appropriate understanding of algebra, as well as skill in applying and communicating it, he or she must develop an ability to read and interpret symbols that represent algebraic ideas and information. In other words, the student needs to learn to scrutinize symbols in algebraic expressions and properly interpret them to understand and work with the ideas and information the symbols convey. Accordingly, we strongly urge the student to make every effort to develop proficiency in reading and writing the "language" of algebra.

Consider a general algebraic statement, a^m, where a is a real number, and m is a positive integer. (That is, $a \in \mathrm{R}$, and $m \in \mathrm{N}$.) By definition, $a^m = a \cdot a \cdot a \cdot a \cdot a$ ... etc. … etc. … for a total of m factors.

Most likely, this is not new to you. We want you to focus on what the symbols convey. Numbers multiplied together to produce a product are called <u>factors</u> of the product. For any real number a, a^m is the product of m of the a's lined up and multiplied together. Notice that the number of a's lined up in the product, i.e. the number of factors could be 1, or 2, or 3, or 4, or 5, or 6, … or 25, … or 200 … or any whole number greater than zero. In this definition, while a is any real number, m is a type of real number that is a natural number, i.e. m is a positive integer.

To be sure we're clear, note that

$a^2 = a \cdot a, \quad a^3 = a \cdot a \cdot a, \quad a^6 = a \cdot a \cdot a \cdot a \cdot a \cdot a, \quad \text{and} \quad a^8 = a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a.$

What about the case in which $m = 1$? Well that's just a^1 which, in accordance with our definition, will be a.

For our expression a^m, a is called the **base**, and m is called the **exponent**. When we read a^m, we can think of it as "the m^{th} power of a," or as "a raised to the m^{th} power," or, as we sometimes say, "a to the m^{th}." Note that we can use any letter we wish for the base as long as we're consistent with the definition we've developed. Moreover, the base does not have to be just a single letter (or literal number). The base can be a simple or elaborate algebraic expression.

B. From Chapter 2, pages 57-58.

$3y^2 = 3 \cdot y \cdot y$, while $(3y)^2 = (3y)(3y) = (3 \cdot 3)(y \cdot y) = 9y^2$.

Notice how parentheses serve to identify the base. The base is the factor immediately to the left and below the exponent.

$(-17)^2 = (-17)(-17) = 289$. The base is minus 17.

$-17^2 = -289$. The base is 17. The exponent refers only to the 17.

-17^2 is $(-1)(17)^2$.

$3(y+6)^2 = 3(y+6)(y+6)$. The base, regarding the exponent 2, is $(y+6)$.

$(3y+6)^2 = (3y+6)(3y+6)$. The base is $3y+6$.

$(x-3)^5 = (x-3)(x-3)(x-3)(x-3)(x-3)$. The base is $x-3$.

Incidentally, this is **not** the same quantity as $(x^5 - 3^5)$. We can easily verify this. For example, if we let $x = 1$, then $(x-3)^5 = (1-3)^5 = (-2)^5 = -32$.

But if $x = 1$, $(x^5 - 3^5) = (1^5 - 243) = 1 - 243 = -242$.

The concept of a base and its exponent provides a kind of shorthand notation that's very useful. It simplifies algebraic expressions. For example, as you may recall, the speed of light in a vacuum is about 3.00 X 10^8 meters per second (m/sec). If we didn't use exponents, we'd likely write this speed as 3 multiplied by eight factors of 10, i.e. 300,000,000 m/sec. If you think that's not terribly laborious, we'll agree.

How about this example? Consider Avogadro's number, 6.02 X 10^{23} molecules per mole. You may recall this from previous course work in science. A mole of a chemical compound is an amount of the substance having mass equal to its molecular weight in grams. Avogadro determined that a mole of any chemical compound has approximately 6.02 X 10^{23} molecules of the substance. For example, the molecular weight of water (H_2O) is 18, so 18 grams of pure water (a little more than half an ounce) would contain about 6.02 X 10^{23} molecules. Without the use of our exponent notation, we'd have to write it out by taking 6.02 and moving the decimal point 23 places to the right. The result would be

$$602{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ molecules/mole.}$$

You can see the advantage of our exponent notation, a^m where a is a real number, and m is a positive integer.

C. From Chapter 2, pages 64-66.

Now we can go a step further. Consider the expression $\frac{1}{a^{n-m}}$ which resulted from $\frac{a^m}{a^n}$ when $n > m$. We're going to multiply $\frac{1}{a^{n-m}}$ by the number 1. You'll agree that any number times 1 is the same number in our system of numbers, i.e.

$3\cdot 1=3,\ \ x\cdot 1=x,\ \ (2a+3b^3)(1)=(2a+3b^3),\ \ \left|\frac{1}{x^3}\right|(1)=\left|\frac{1}{x^3}\right|$ and so on. If we write the number 1 as $\frac{a^{m-n}}{a^{m-n}},\ \ a\neq 0,$ and then use that as the multiplier, we have

$$\frac{1}{a^{n-m}}=\left|\frac{1}{a^{n-m}}\right|(1)=\left|\frac{1}{a^{n-m}}\right|\left|\frac{a^{m-n}}{a^{m-n}}\right|=\frac{a^{m-n}}{a^{(n-m)+(m-n)}}=\frac{a^{m-n}}{a^0}=\frac{a^{m-n}}{1}=a^{m-n}.$$

Notice we've concluded that $\frac{1}{a^{n-m}}=\frac{a^{m-n}}{1}$. The exponent $n-m$, is the negative of $m-n$, i.e. $(n-m)=(-1)(m-n)$. Suppose $n-m=2$. Then we would have $m-n=-2$, and $\frac{1}{a^2}=\frac{a^{-2}}{1}$. We can conclude that for any non-zero number, a. and any whole number exponent, k, $a^k=\frac{1}{a^{-k}}$ and $a^{-k}=\frac{1}{a^k}$.

So if you have a factor in the numerator of a fraction, you can rewrite it as a factor in the denominator by replacing the exponent with its opposite, i.e. by changing the algebraic sign of the exponent. You can rewrite a denominator factor as a factor in the numerator in the same way. Remember, this **applies to factors, not to portions of a sum or difference.**

Here are some examples. (It is understood that denominators never have a numerical value of zero.)

11. $x^3=\frac{1}{x^{-3}}$.

12. $y^{-2}=\frac{1}{y^2}$.

13. $\dfrac{3x^2}{y^4} = 3x^2 y^{-4}$.

14. $\dfrac{a^3b^5c^2}{ab^3} = a^3 \cdot a^{-1} \cdot b^5 \cdot b^{-3} \cdot c^2 = a^2b^2c^2$.

15. $\dfrac{4x^5}{x(2x+1)} = \dfrac{4x^5 \cdot x^{-1}}{(2x+1)} = \dfrac{4x^4}{2x+1}$.

16. $(3y)^{-3} = \dfrac{1}{(3y)^3}$.

17. $\dfrac{p^4(p+q)^{-2}}{7p^4} = \dfrac{(p^4)(p^{-4})}{7(p+q)^2} = \dfrac{p^0}{7(p+q)^2} = \dfrac{1}{7(p+q)^2}$.

18. $\dfrac{r^3s^{-5}t^{-4}}{r^{-2}s^2t^5} = \dfrac{r^3 \cdot r^2}{s^2 \cdot s^5 \cdot t^5 \cdot t^4} = \dfrac{r^5}{s^7 \cdot t^9}$.

D. From Chapter 3, Polynomials: Operations and Factoring, pages 103-105.

A **polynomial is a type of algebraic expression** that is very common and useful in various applications of algebra. Functions and equations used in science, engineering, and the behavioral and social sciences often involve polynomials.

First, we'll define " monomial." A **monomial** is a **product** of a number and one or more variables, or it could be just a number. A key point is that the **exponents of any variables in the product must be whole numbers** for the product to be a monomial. You can say a monomial is a term in which any exponents of variables are whole numbers (i.e. zero, or positive integers). Examples of monomials are $3x^2y^5$, $7x^3$, $-4xy$, $\sqrt{2}z^5$ and 9. Note that "9" is a monomial since $9 = 9x^0$ and $x^0 = 1$. Examples of terms which are not monomials are

$6x^{2/3}$, $4\sqrt{3y^7}$, $5x^{-2}y^3$, and $\frac{12}{7z}$. In these examples, the condition that the exponent of every variable is a whole number is not met. Note that $4\sqrt{3y^7} = 4\left(\sqrt{3}\right)y^{7/2}$, and $\frac{12}{7z} = \left(\frac{12}{7}\right)\left(z^{-1}\right)$.

Now **a polynomial is a sum of monomials**. Accordingly, a polynomial can be a real number, or it can have one or more variables. A polynomial which is the sum of two monomials is a **binomial**. If it consists of three monomials added together, it is a **trinomial**.

If a polynomial has one variable, its **degree** is the numerical value of the largest exponent that appears in the polynomial. For example, if the polynomial is $5x^4 + 3x^2 - 2x + 7$, its degree is four. The monomials in a polynomial are called **terms** of the polynomial. If a polynomial has more than one variable, its **degree** will be the **sum of the exponents of the term having the largest sum of exponents**. For example, the polynomial $6x^4y^3 + 3xy^4 - 2x^2y$ has a degree of seven and $8y^4z^2 - 10x^3y - 3xz$ has a degree of six. If a polynomial is just a real number other than zero, then its degree is zero. If the polynomial is zero, then it has no degree.

Again, a polynomial consists of one or more monomials added together. In any term of the polynomial, the number that multiplies the variable, or product of variables is called the **coefficient** of the term. Regarding the two polynomials cited near the end of the paragraph immediately above, the coefficients of the terms are, respectively, $6,\ 3,\ -2$ and $8,\ -10,\ -3$.

According to commonly used format, the first term of a **single-variable polynomial** is the term with the largest exponent. The remaining terms are listed in decreasing order of exponents. This is a standard format. For example, we would write $-2x^3 + 9x^2 - 25x + 12$ rather than the same polynomial with the terms in a different order, even though the two arrangements would be mathematically equivalent. In this format, the coefficient of the first term is called the **leading coefficient**. The smallest exponent that a variable could have in a polynomial term is zero. With only one variable, the coefficient of this term is called the **constant term**. This term will just be a number, i.e. a constant. In the example just given, the leading coefficient is minus two and the constant term is 12. Note that this polynomial can be seen as $(-2)x^3 + 9x^2 + (-25)x^1 + 12x^0$.

If a polynomial has two or more variables, then good format would suggest the first term be the term in which the sum of the exponents is greatest, with subsequent terms listed in order of decreasing sums of exponents. Accordingly, terms of a polynomial in variables r and s would be arranged as follows:

$$23r^5s^6 - 12r^4s^2 + 6r^3s - 5rs.$$

If two of the terms had exponent sums that were equal, they could be listed next to each other in which ever order was most appealing. This is just a matter of format, and, as you know, the order of terms does not affect the numerical value of the polynomial for any particular values of its variables.

The concepts of a leading coefficient and constant term are useful and of interest when we're dealing with single-variable polynomials. The standard form of an n^{th} degree polynomial in x, say, is written as

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \quad \dots \quad + a_2x^2 + a_1x + a_0.$$

The coefficients are real numbers and n is a positive integer. Note that the last two terms are really $a_1x^1 + a_0x^0$ since $a_1x = a_1x^1$ and $a_0 = a_0 \cdot 1 = a_0x^0$.

E. From Chapter 4, Operations With Algebraic Fractions, pages 146-147.

As with multiplication of arithmetic fractions, multiplication of algebraic fractions is straight forward. You just multiply the numerators and multiply the denominators to obtain, respectively, the numerator and denominator of the product, which you present in simplest terms. Some examples follow.

$$\frac{3}{8}\cdot\frac{12}{5}=\frac{(3)(12)}{(8)(5)}=\frac{36}{40}=\frac{4(9)}{4(10)}=\frac{4}{4}\cdot\frac{9}{10}=1\cdot\frac{9}{10}=\frac{9}{10}. \qquad 3\tfrac{3}{4}\cdot 4\tfrac{1}{2}=\frac{15}{4}\cdot\frac{9}{2}=\frac{135}{8}.$$

$$\frac{y-2}{5}\cdot\frac{y^2+2}{y^2-4}=\frac{y-2}{5}\cdot\frac{y^2+2}{(y+2)(y-2)}=\frac{(y-2)(y^2+2)}{5(y+2)(y-2)}=\frac{(y-2)(y^2+2)}{(y-2)(5)(y+2)}$$

$$=\frac{(y-2)}{(y-2)}\cdot\frac{(y^2+2)}{5(y+2)}=1\cdot\frac{y^2+2}{5(y+2)}=\frac{y^2+2}{5(y+2)}.$$

Notice in the first example above, people will say the 4's cancel. That's fine. However, it's important that you understand that we are able to divide out the 4's, i.e. cancel them, because 4 is a factor in both the numerator and the denominator. We can then write $\frac{4}{4}$ as a factor of the rest of the fraction. Since $\frac{4}{4}=1$, we replace that factor with 1 to reduce the fraction to simplest terms. **You can cancel numbers or algebraic expressions only if they are common factors in the numerator and denominator of a fraction.**

In the second example, the resulting fraction $\frac{135}{8}$ cannot be reduced. As you may know, with rational numbers, if you write the numerator and denominator as a product of powers of prime factors, you can tell if there are any common factors. In this case, $135=5\cdot 27=5\cdot 3^3$ and $8=2^3$, so $\frac{135}{8}=\frac{3^3\cdot 5}{2^3}$. Accordingly, there are no common factors in the numerator and denominator and the fraction is already in simplest terms. (The only divisors of a prime number are itself and one; 2, 3, and 5 are prime numbers.)

In the third example, the multiplier, multiplicand and product are algebraic fractions. In multiplying algebraic fractions, you should write the numerator and denominator of the product in factored form if the corresponding algebraic expressions are factorable. Often, it will be helpful to factor algebraic expressions in the multiplier and multiplicand as well. In our example, y^2-4 was factored into $(y+2)(y-2)$ and $y-2$ emerged as a common factor in the numerator and denominator which we could divide out.

Examples. We'll do several more examples to illustrate the concepts. We'll show more step-by-step detail than one would ordinarily write.

1. $$\frac{12z^2}{y^2}\cdot\frac{(-3y)}{8z}=\frac{(12z^2)(-3y)}{8y^2z}=\frac{(4)(3)(-3)\cdot y\cdot z\cdot z}{(4)(2)\cdot y\cdot y\cdot z}$$
$$=\frac{4}{4}\cdot\frac{y}{y}\cdot\frac{z}{z}\cdot\frac{-9z}{2y}=1\cdot1\cdot1\cdot\frac{-9z}{2y}=-\frac{9z}{2y}.$$

2. $$\frac{(6y^2+17y-3)}{7y}\cdot\frac{(y^2+y-12)}{y^2-9}=\frac{(6y-1)(y+3)}{7y}\cdot\frac{(y-3)(y+4)}{(y+3)(y-3)}$$
$$=\frac{(6y-1)(y+3)(y-3)(y+4)}{7y(y+3)(y-3)}$$
$$=\frac{(6y-1)(y+4)}{7y}\cdot\frac{(y+3)}{(y+3)}\cdot\frac{(y-3)}{(y-3)}$$
$$=\frac{(6y-1)(y+4)}{7y}\cdot1\cdot1=\frac{(6y-1)(y+4)}{7y}.$$

3. $$\frac{5}{(x^2-x-6)}\cdot\frac{(3-x)}{(5x+15)}=\frac{5}{(x-3)(x+2)}\cdot\frac{(-1)(x-3)}{5(x+3)}=\frac{(-5)(x-3)}{(5)(x-3)(x+2)(x+3)}$$
$$=\frac{(-5)}{(5)}\cdot\frac{(x-3)}{(x-3)}\cdot\frac{1}{(x+2)(x+3)}=-\frac{1}{(x+2)(x+3)}.$$

F. From Chapter 4, pages 148-149.

As you know, a quotient consists of a dividend divided by a non-zero divisor. **Division of two fractions is accomplished by multiplying the dividend by the**

reciprocal of the divisor. Replacing a divisor by its reciprocal is often referred to as **"inverting the divisor."**

If $\frac{a}{b}$ and $\frac{c}{d}$ are algebraic fractions, and $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. This is a rule readily recalled and easily employed. It's better to understand how it is derived, rather than to just "blindly" memorize it.

Consider the quotient of algebraic fractions $\frac{a/b}{c/d}$. Clearly, $\frac{a/b}{c/d} \cdot 1 = \frac{a/b}{c/d}$. The strategy is to write the number 1 in a way that will be useful. Note that the reciprocal of c/d is d/c. If we write 1 as $\frac{d/c}{d/c}$ and substitute the latter for 1 in the above equation, we will have

$$\frac{a/b}{c/d} = \frac{a/b}{c/d} \cdot 1 = \frac{a/b}{c/d} \cdot \frac{d/c}{d/c} = \frac{(a/b)(d/c)}{(c/d)(d/c)} = \frac{(a/b)(d/c)}{cd/dc} = \frac{(a/b)(d/c)}{cd/cd} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}.$$

We conclude that $\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$, hence the rule "To divide fractions, invert the divisor and multiply." This strategy of multiplying by 1, with 1 written in a fruitful way, is often very useful in manipulating algebraic expressions. As indicated earlier, it is often helpful to write numerators and denominators of multipliers and multiplicands in factored form. Finally, the quotient should be written in simplest terms.

Examples.

4. $\frac{3}{8} \div \frac{7}{12} = \frac{3}{8} \cdot \frac{12}{7} = \frac{36}{56} = \frac{(4)(9)}{(8)(7)} = \frac{(4)(9)}{(4)(2)(7)} = \frac{9}{14}$.

5. $$\frac{15/17}{5} = \frac{15/17}{5/1} = \frac{15}{17}\cdot\frac{1}{5} = \frac{(5)(3)}{(5)(17)} = \frac{5}{5}\cdot\frac{3}{17} = \frac{3}{17}.$$

6. $$\frac{3(x+y)}{1/(x+y)} = \frac{3(x+y)}{1}\cdot\frac{(x+y)}{1} = 3(x+y)^2.$$

7. $$\frac{\frac{5p^2q^3}{3}}{\frac{15pq}{27}} = \frac{5p^2q^3}{3}\cdot\frac{27}{15pq} = \frac{5(27)p^2q^3}{3(15)pq} = \frac{5(3^3)p^2q^3}{3^2(5)pq} = \frac{5}{5}\cdot\frac{3^2}{3^2}\cdot\frac{pq}{pq}\cdot\frac{3pq^2}{1} = 3pq^2.$$

8. $$\frac{9x^2+24x+16}{25x^2-16} \div \frac{6x+8}{5x+4} = \frac{(3x+4)^2}{(5x-4)(5x+4)} \div \frac{2(3x+4)}{(5x+4)}$$

$$= \frac{(3x+4)^2}{(5x-4)(5x+4)}\cdot\frac{(5x+4)}{2(3x+4)} = \frac{(3x+4)(3x+4)(5x+4)}{2(3x+4)(5x-4)(5x+4)}$$

$$= \frac{(3x+4)}{(3x+4)}\cdot\frac{(5x+4)}{(5x+4)}\cdot\frac{3x+4}{2(5x-4)} = 1\cdot 1\cdot\frac{3x+4}{2(5x-4)} = \frac{3x+4}{2(5x-4)}.$$

9. $$\frac{x^3-8y^3}{4x+4y} \div \frac{x-2y}{x+y} = \frac{x^3-8y^3}{4(x+y)}\cdot\frac{x+y}{x-2y} = \frac{(x-2y)(x^2+2xy+4y^2)}{4(x+y)}\cdot\frac{(x+y)}{(x-2y)}$$

$$= \frac{(x-2y)}{(x-2y)}\cdot\frac{(x+y)}{(x+y)}\cdot\frac{x^2+2xy+4y^2}{4} = \frac{x^2+2xy+4y^2}{4}.$$

10. $$\frac{\frac{6}{x^2-4}}{\frac{18}{x-2}} = \frac{6}{x^2-4}\cdot\frac{x-2}{18} = \frac{6(x-2)}{18(x+2)(x-2)} = \frac{6}{18}\cdot\frac{(x-2)}{(x-2)}\cdot\frac{1}{(x+2)} = \frac{1}{3(x+2)}.$$

G. From Chapter 6, Solving Inequalities, pages 262-263.

Examples. Find the solution set for each of the following inequalities.

1. $5(x-2) > 3x-2(x+3)$.

$5(x-2) > 3x-2(x+3)$

$5x-10 > 3x-2x-6$ Cleared parentheses.

$5x-10 > x-6$ Combined like terms.

$5x-x-10 > -6$ Subtracted x from both sides.

$4x-10 > -6$ Combined like terms.

$4x > -6+10$ Added 10 to both sides.

$4x > 4$ Combined like terms.

$\frac{4x}{4} > \frac{4}{4}$ Divided both sides by 4.

$x > 1.$ Simplified expressions.

$S = \{x \mid x > 1\}$. We can say $S = (1, \infty)$.

Note that we can't check the roots because there's an unlimited number of them. However, we can verify that certain values are consistent with the solution. For example, if $x = 1$, we have $5(1-2) = 5(-1) = -5$ and $3(1)-2(1+3) = 3-2(4) = -5$. When $x = 1$, $5(x-2) = 3x-2(x+3)$. This implies $x = 1$ is not a solution. You can test values greater than one to verify that they are solutions. You can also check values less than one to verify that they do not belong to the solution set. For example, $5(0-2) = -10 \not> 3(0)-2(0+3) = -6$. Figure 3.2 depicts the solution set.

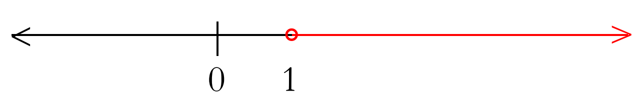

Fig. 3.2 $\boldsymbol{x > 1}$

2. $5y + 7 \le 15(y - 3)$.

$5y + 7 \le 15(y - 3)$	
$5y + 7 \le 15y - 45$	Cleared parentheses.
$5y - 15y + 7 \le -45$	Subtracted $15y$ from both sides.
$5y - 15y \le -45 - 7$	Subtracted 7 from both sides.
$-10y \le -52$	Combined like terms.
$\frac{-10y}{-10} \ge \frac{-52}{-10}$	Divided both sides by -10.
$y \ge 5.2$	Simplified expressions.

$S = \{y \mid y \ge 5.2\}$, i.e. $S = [5.2, \infty)$.

Note the order reversal when both sides were divided by minus 10. Figure 3.3 is a graph of the solution set.

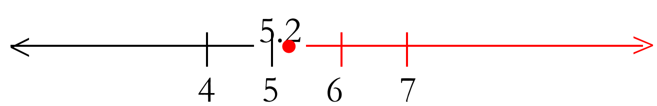

Fig. 3.3 $y \ge 5.2$

H. From Chapter 8, Graphing Linear Equations, page 354.

With some (hopefully) interesting algebra, we can derive another form of the equation of the line called **the slope-intercept** form. This will be very useful for plotting graphs and for determining equations of lines in graphs.

$$Ax + By = C \quad \Rightarrow \quad By = -Ax + C \quad \Rightarrow \quad y = -\frac{A}{B}x + \frac{C}{B}. \qquad (1)$$

If $x = 0$, $y = \frac{C}{B}$. The line crosses the y axis at $(0, \frac{C}{B})$. $\therefore$ $\frac{C}{B}$ is the y-intercept.

Every point on the line satisfies $Ax + By = C$. Suppose (x_1, y_1) and (x_2, y_2) are points on the line. Then

$Ax_1 + By_1 = C$ (2) and

$Ax_2 + By_2 = C.$ (3)

Equation (3) $-$ Equation (2) gives us

$Ax_2 + By_2 - (Ax_1 + By_1) = C - C = 0.$

$\therefore \quad Ax_2 + By_2 - Ax_1 - By_1 = 0 \quad \Rightarrow \quad Ax_2 + By_2 = Ax_1 + By_1$

$\Rightarrow \quad Ax_2 - Ax_1 = A(x_2 - x_1) = By_1 - By_2 = B(y_1 - y_2) = -B(y_2 - y_1)$

$$\Rightarrow \quad \frac{y_2 - y_1}{x_2 - x_2} = -\frac{A}{B}. \quad \therefore \quad \text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{A}{B}.$$

Referring to Equation 1, above, we have

$y = -\frac{A}{B}x + \frac{C}{B}$. Let $b = \frac{C}{B}$, the ordinate of the y-intercept. Since $m = -\frac{A}{B}$, we have $y = mx + b$ where m is the slope and b is the y value of the y-intercept.

$y = mx + b$ is the slope-intercept form of the equation of a line.

I. From Chapter 10, Functions, pages 466-469.

Functions are rules or specifications for processing numbers. For example, the equation $y = 2x - 3$ can be thought of as a rule that says "For each numerical value assigned to x, multiply it by 2 and subtract 3 to obtain a corresponding numerical value of *y*." By this rule, or numerical procedure, if $x=1$, $y=2(1)-3=2-3=-1$. If $x=10$, $y=2(10)-3=20-3=17$, and if $x=-6$, $y=2(-6)-3=-12-3=-15$. We say y is a function of *x* and we indicate this by writing $y=f(x)$ where *f* is the function or rule that is to be applied. In this example, the rule or procedure that *f* represents is to multiply by 2 and subtract 3. It does it to any number properly assigned to *x* to produce a numerical value of *y*. **In order for *f* to be a function, for any numerical value assigned to *x*, there must be one, and only one corresponding numerical value of *y*.**

In the case of $y=2x-3$, *x* can be any real number, and for any real number assigned to *x*, there will be one, and only one real number *y*. The set of numbers that can be assigned to *x* is called the **domain** of the function, and the set of numbers that are corresponding *y*-values is the function's **range**. As you know, if x and y take on various numerical values, they are **variables**. If y represents a function of x, i.e. if $y=f(x)$, then y is the **dependent variable** and *x* is called the **independent variable**. You can see the motivation for assigning these names to the variables. The variable, *x* can be assigned any number in the set called the domain, but the numerical value of *y* depends upon the value assigned to *x*.

Another example of a function is $y=3\sqrt{x}$. We could use a different letter for the function and say $y=g(x)$ where $g(x)=3\sqrt{x}$. Here, the function g prescribes that for any numerical value that we assign to x, we are to find its positive square root, and multiply that root by 3. Accordingly, if $x=4$, $y=3\sqrt{4}=3(2)=6$, and if $x=16$, $y=3\sqrt{16}=3(4)=12$. Also, if $x=400$, $y=3\sqrt{400}=3(20)=60$. Note an important restriction in this example. Our independent variable, *x*, cannot be negative because if it were, we'd have the square root of a negative number, and that square root is not a real number—it's imaginary. So for this function, $x\geq 0$. Therefore the domain of $g(x)$ is the set of non-negative real numbers. The smallest value of x in the range of this function is zero. When $x=0$, $y=3\sqrt{0}=3(0)=0$. Therefore, the range is the set of non-negative real numbers. (The range is not the set of positive real numbers, because zero is in the range, and zero is neither positive nor negative.)

Another way to consider functions is to think in terms of **input** and **output**. The independent variable is thought of as the **input variable** and the dependent variable is the **output variable**. For the equation $y = f(x) = 5x - 6$, the independent variable, x, is the input variable, and y, i.e. $f(x)$, is the dependent, or output variable. The set of all possible input values, the domain of the function, is the set of real numbers. For any real number, x, in processing it, the function f requires that you multiply it by 5 and then subtract 6 to get the corresponding output value. For example,

If $x = 20$, $y = f(x) = 5(20) - 6 = 100 - 6 = 94$. Input $= 20$; output $= 94$.

If $x = -50$, $y = f(x) = 5(-50) - 6 = -250 - 6 = -256$. Input $= -50$; output $= -256$.

If $x = \frac{3}{17}$, $y = f(x) = 5\left(\frac{3}{17}\right) - 6 = \frac{15}{17} - 6 = -5\frac{2}{17}$. Input $= \frac{3}{17}$; output $= -5\frac{2}{17}$.

If $x = \sqrt{3}$, $y = f(x) = 5\left(\sqrt{3}\right) - 6 = 5\sqrt{3} - 6$. Input $= \sqrt{3}$; output $= 5\sqrt{3} - 6$.

It should be clear that, for the function $f(x) = 5x - 6$, the set of all possible output values, the range, is also the set of real numbers.

The old adage that "A picture is worth a thousand words," is often somewhat true. Figures 1.1, and 1.2, below, illustrate the concept of a function. Figure 1.1 shows a Venn diagram of two sets, **X**, a set of three elements constituting the domain of a function, and **Y**, a set of three corresponding elements, the range of the same function. The arrows show that when the function is applied to each domain element, i.e. to each *x,* one, and only one corresponding range element results. We see x_1, as input to the function, produces y_1 as output. Similarly, x_2 produces y_2, and x_3 input gives y_3 output. There is a correspondence between the elements of the two sets. For each element of set **X**, there is a corresponding element of set **Y**. This correspondence is an example of a **relation**. When you have a relation in which every element of the first set corresponds with **one, and only one** element of the second set, then you have a **function.**

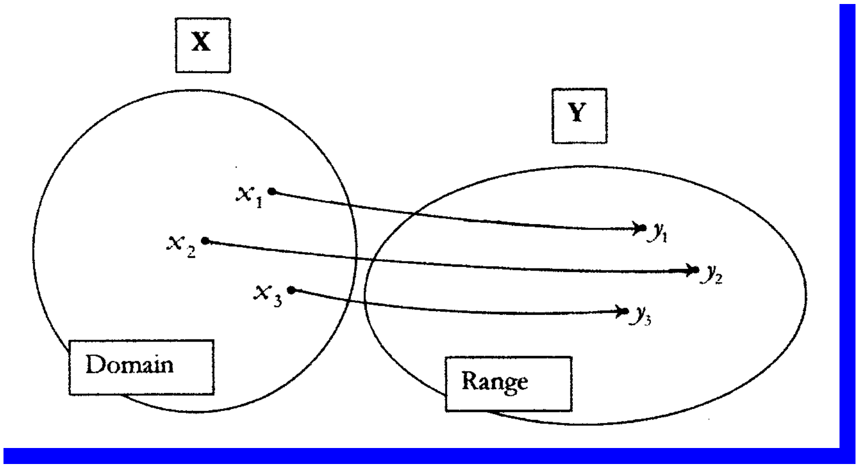

Fig. 1.1 The Function, A Special Kind of Relation

Now, if more than one element of the domain, **X**, corresponds to an element of the range, **Y**, we still have a function because we meet the condition that for each element of **X,** there is one, and only one element of **Y.** However, if any element of **X** corresponds with more than one element of **Y**, then you have a relation that is <u>not</u> a function. Figure 1.2, below, illustrates these two situations.

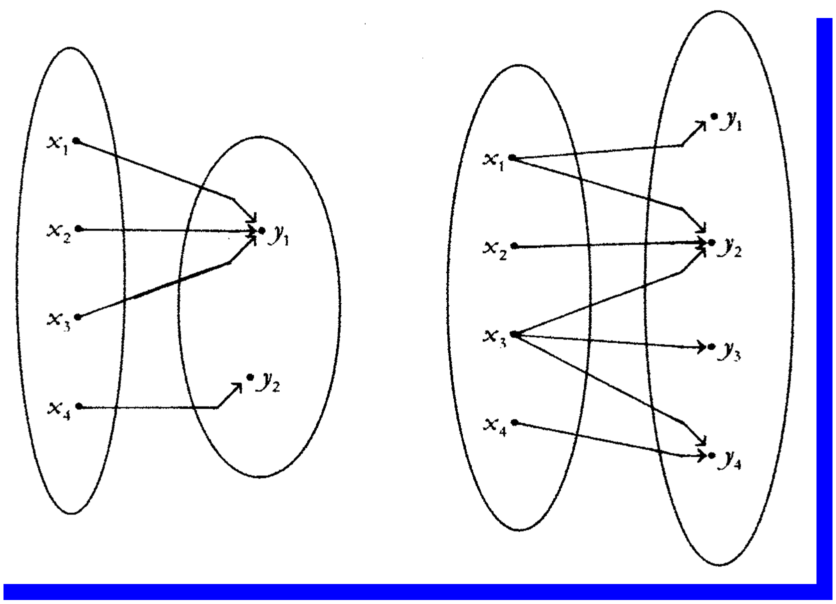

Fig. 1.2 A Function (Left), and A Non-Function Relation (Right)

Notice in the two sets at the left (above), every x value corresponds to one, and only one y value. So that pair of sets illustrates a function. In the pair of sets at the right, every x corresponds to at least one y, **but at least one *x* corresponds to more than one *y*.** In fact, two of them do. Note that x_1 corresponds to two values, y_1 and y_2, and x_3 corresponds to three y-values, y_2, y_3, and y_4. Therefore, the two sets at the right represent a relation, but they do not represent a function.